

GEAR REGS PROJECTS SUMMER 2015

| | Project Supervisor | Project Title | Project Location | Project Dates | Project Description | # of Positions Available | More Information |
|----|--|---|--------------------------------|----------------------------------|--|--------------------------|---|
| 1. | Jean-Marc Schlenker jean-marc.schlenker@uni.lu | Renormalized volume of quasifuchsian manifolds with particles | University of Luxembourg | May 5, 2015 to July 4, 2015 | <p>Although the volume of any quasifuchsian manifold M is infinite, one can define a "renormalized" volume $V_R(M)$ which is finite. It is known to have a number of remarkable properties, for instance (1) it provides a Kähler potential for the Weil-Petersson metric on the Teichmüller space of each boundary component of M, (2) it differs from the volume of the convex core by a bounded amount, (3) it is bounded by a constant times the Weil-Petersson distance between the two conformal metrics at infinity.</p> <p>This renormalized volume seems useful in understanding the geometry of 3-manifolds that fiber over the circle and Weil-Petersson geometry.</p> <p>The first goal of the project would be to extend this notion to quasifuchsian manifolds with "particles", that is, cone singularities of angle less than π along lines connecting the boundaries at infinity. Since a Bers double uniformization theorem is known for those manifolds with particles, the renormalized volume could be a tool for understanding the Weil-Petersson metric of hyperbolic metrics with cone singularities of fixed angle, which is known to be Kähler (Schumacher-Trapani).</p> <p>A second goal could be to bound the difference between the renormalized volume and the volume of the convex core in this case "with particles".</p> <p>A third, and probably more difficult goal, would be to find an analog of (3) above. Here new ideas would be necessary since the proof in the non-singular setting uses a Nehari estimate which is not available when particles are present.</p> | 1 | http://math.uni.lu/schlenker |
| 2. | Marco Gualtieri and Motohico Mulase mgualt@math.toronto.edu | WKB, topological recursion, and spectral networks | University of California Davis | July 13, 2015 to August 23, 2015 | The main purpose of the project is to understand how topological recursion and quantum curves relates to the current study of the exact WKB method of Aoki, Kawai et al, as well as the spectral networks of Gaiotto-Moore-Neitzke. | 1 | N/A |

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| 3. | Anna Wienhard wienhard@uni-heidelberg.de | Deformations of convex real projective structures | University of Heidelberg | June 22, 2015 to July 10, 2015 | <p>The aim of this project is to give geometric description of the possible ways a convex real projective structure on a surface can diverge. As a first step, one should consider the deformation space of convex real projective structures on a pair of pants. Choosing an ideal triangulation of this pair of pants into two ideal triangles, the work of Fock-Goncharov implies that this space can be parameterized by specifying a ``twist'' parameter and a ``bulge'' parameter for each edge, and a ``triangle parameter'' for each triangle.</p> <p>If the eigenvalues for the holonomy about the boundary components of the pair of pants are fixed, then the twist parameters for the three edges of the ideal triangulation are determined. This condition also gives a pair of simple relations between the bulge parameters associated to the three edges, and another simple relation between the triangle parameters (see Tengren Zhang's work). Hence, the space of such convex real projective structures on the pair of pants is a two-dimensional space that is parameterized by a bulge parameter and a triangle parameter.</p> <p>These geometric parameters also allow to qualitatively describe how the image of the developing map of the convex real projective structure degenerates in this two-dimensional deformation space.</p> <p>In this project this description shall be used to describe 1) the local behavior of lengths of curves, 2) to identify geometric limiting objects for deformations of the real convex projective structure.</p> | 1 | N/A |

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| 4. | Sean Lawton, Chris Manon slawton3@gmu.edu | Experiments with Character Varieties | George Mason University | June 15, 2015 - August 15, 2015 | <p>For a discrete group D and a Lie group G, the character variety $X(D, G)$ is the moduli space of representations in $\text{Hom}(D, G)$. Character varieties are important in describing how invariants of a topological space M can constrain the types of geometry it can support.</p> <p>The purpose of this project is to experimentally investigate the structure of character varieties when $D=F$ is a finite rank free group and $G=SL(n, \mathbb{C})$.</p> <p>Horowitz Words and Generalizations: Horowitz pairs u, v in a free group F are non-conjugate words yet for all $[f]$ in $X(F, SL(n, \mathbb{C}))$ the identity $\text{Tr}(f(u))=\text{Tr}(f(v))$ holds. Such pairs are only known to exist when $n=2$. In this project we will work towards classifying these pairs for $n=2$, while also searching for examples when $n=3$.</p> <p>Arithmetic Mapping Class Dynamics: For a free group F, the outer automorphisms $\text{Out}(F)$ acts on $X(F, SL(n, \mathbb{C}))$ rationally. For large enough primes p this action descends to an action on the $(\mathbb{Z}/p\mathbb{Z})$-points of the scheme associated to $X(F, SL(n, \mathbb{C}))$. For this project, we will explore the orbit structure under mapping classes for the $n=2$ case looking for patterns as p varies.</p> <p>Okounkov Bodies: For an algebraic variety X an Okounkov body is a convex polyhedron which can be used to combinatorially control the coordinate ring $\mathbb{C}[X]$. Recently methods have been developed to construct Okounkov bodies for character varieties $X(F, SL(n, \mathbb{C}))$, but many of the combinatorial features of these polyhedra remain mysterious. We will work to compute the face poset of the Okounkov bodies for low rank free groups and $n=2, 3$.</p> <p>Tropical Varieties: A tropical variety of an algebraic variety is a weighted polyhedral fan which encodes algebraic and geometric characteristics combinatorially. Using software like Gfan, we will investigate the face structure of the tropical varieties of some simple classes of character varieties, and compute their weights.</p> | 1-4 | http://math.gmu.edu/~slawton3/ |
| 5. | Vincent Delecroix vincent.delecroix@labri.fr | Gap distribution in Prym locii | Marseille and Bordeaux | July 11, 2015 - August 15, 2015 | <p>There are $SL(2, \mathbb{R})$-invariant locii of intermediate dimension in the stratum $H(1, 1)$ of translation surfaces called Prym locii or real multiplication locii (McMullen). The aim of the project is to understand the gap distribution of saddle connections on some (or possibly all) these intermediate locii.</p> <p>On an experimental side, we propose to design and implement an algorithm to compute efficiently and exactly the set of saddle connections of given length on a square tiled surfaces. This algorithm can be used to give some light on the expected answer for real multiplication locii when the discriminant is a square.</p> | 1 | N/A |