GEAR Junior Retreat – Abstracts

July 23 – August 3, 2012
University of Illinois at Urbana-Champaign

All talks will be in 314 Altgeld Hall

Jayadev Athreya: *Quantitative Aspects of Translation Surfaces* (mini-course)

Monday 7/30 – 9:00am; Wednesday 8/1 – 1:00pm; Friday 8/3 – 9:00am

Abstract: We'll study dynamics of the $\mathrm{SL}(2,\mathbb{R})$ action on moduli spaces of translation surfaces, and show how quantitative aspects of this dynamics can yield quantitative information about dynamics on individual translation surfaces, and in particular information for rational polygonal billiards. We will build on earlier work of Masur, Masur-Smillie, and Kerckhoff-Masur-Smillie, and try and discuss more recent work of Avila, Eskin, Gouezel, Forni, Masur, Yoccoz, Zorich, and others.

Sam Ballas: *Twisted Cohomology and Deformation Theory*

Tuesday 7/24 – 3:15pm

Abstract: Up to conjugacy, local deformations of a lattice in a lie group are controlled up to first order by the twisted cohomology. In this talk I will explain this connection and show some examples of how one can hope to understand the local structure of certain character varieties by studying cohomology.

Anja Bankovic: *Equivalent curves in surfaces*

Friday 8/3 – 4:30pm

Abstract: In this talk we'll discuss length functions on a variety of spaces of metrics on a genus $g > 1$ surface. In particular, we will examine a phenomenon first observed by Horowitz and Randol that there are arbitrary large sets of homotopy classes of closed curves on a surface whose lengths are equal in every hyperbolic metrics. We will consider the family of nonpositively curved singular flat metrics and show that this phenomenon persists in a large subfamily of flat metrics coming from q-differentials. However in the limit the phenomenon disappears.

Yves Benoist: *On divisible convex sets* (mini-course)

Monday 7/23 – 3:15pm; Tuesday 7/24 – 9:00am; Thursday 7/26 – 3:15pm

Abstract: This course is an introduction to the periodic tilings of convex subsets of the real projective space. These tilings generalize the periodic tilings of the real hyperbolic space. The isometry groups of such tilings are discrete groups of matrices which occur naturally in families called "moduli spaces" We will focus on the algebraic properties of these groups, on their moduli spaces and on their asymptotic geometry.

Here is the plan of the talks.

1) Group preserving convex cones. Semisimplicity, Zariski closure.
2) Moduli spaces of convex projective structures: openness, closedness, smoothness.
3) Hilbert geometry: hyperbolicity, Monge-Ampere equation.

Steven Bradlow: *Introduction to Higgs bundles on Riemann surfaces* (mini-course)

Monday 7/23 – 11:00am; Wednesday 7/25 – 9:00am; Friday 7/27 – 9:00am

Abstract: The main goal of this mini course is to answer the questions

(a) What are Higgs bundles?
(b) How are they related to surface group representations?

We will begin with the correspondence between fundamental group representations, local systems, and smooth principal bundles with flat connections. After a brief introduction to holomorphic bundles and their differential geometry we will show how the defining data for a flat principal bundle lead to the idea of a Higgs bundle. We will explore the relation between harmonic metrics on flat bundles and solutions to the Hitchin equations on Higgs bundles. We will describe (without detailed proofs) the main tools in the Higgs bundle toolbox, namely the Hitchin-Kobayashi correspondence for Higgs bundles, the Corlette-Donaldson theorem, and the $\mathbb{C}^*$-action and Hitchin function on the moduli space of Higgs bundles. By considering representations into $\mathrm{SL}(2,\mathbb{R})$ we will explore how quadratic differentials and Teichmüller space appear in the context of Higgs bundles. We will try to ensure that the material is accessible to all members of the GEAR community.

Jeffrey Brock: *Combinatorial Teichmüller Spaces* (mini-course)

Thursday 7/26 – 9:00am; Thursday 7/26 – 1:45pm; Friday 7/27 – 1:45pm

Abstract: In this mini-course, we will discuss the notion of Teichüller space, a parameter space for hyperbolic structures on two dimensional surfaces. After introducing this notion and some of its basic elements we will quickly move to more combinatorial versions of Teichmüller space, realized by considering essential simple closed curves on surfaces and how their geodesic realizations vary with the hyperbolic structure. The combinatorial notions of the 'complex of curves' and the 'pants graph' have played a very important role in improving our understanding of Teichmüller space and its automorphism group. We will introduce these ideas, explain relations between them, and formulate basic connections between the different players in the discussion. Effort will be made to keep the discussion down to earth and elementary, and references will be given for more detailed arguments.

Marc Culler: *Introduction to Hyperbolic 3-manifolds* (mini-course)

Monday 7/23 – 1:45pm; Tuesday 7/24 – 2:45pm; Wednesday 7/25 – 1:00pm

Abstract: The first lecture will discuss examples and basic properties of topological and hyperbolic 3-manifolds, including incompressible surfaces, completeness, injectivity radii, the thick-thin decomposition, structure of ends, and whatever else time permits. The second lecture will be a sketch of the proof of the Mostow Rigidity Theorem and some of its consequences. The third lecture will introduce the $\mathrm{SL}(2,\mathbb{C})$ character variety of a non-compact finite-volume hyperbolic 3-manifold and the relationship between ideal points of the character variety and incompressible surfaces.

Emilio Franco: *Higgs bundles over elliptic curves*

Friday 7/27 – 4:30pm

Abstract: Higgs bundles have been studied mostly over Riemann Surfaces of genus $g > 1$. For $g = 1$, one can prove that a Higgs bundle is (semi)stable if an only if the underlying vector bundle is (semi)stable. This allows us to use Atiyah's classification of vector bundles to describe the moduli space of Higgs bundles over an elliptic curve X as the symmetric product of the cotangent bundle T^*X. We also obtain an explicit description of the Hitchin fibration and we study the generic and non-generic fibres. We extend this study to the structure groups $SL(n,\mathbb{C})$, $\mathrm{PGL}(n,\mathbb{C})$, $\mathrm{Sp}(2m,\mathbb{C})$, $\mathrm{O}(n,\mathbb{C})$, $\mathrm{SO}(n,\mathbb{C})$, $\mathrm{U}(p,q)$, $\mathrm{GL}(n,R)$ and $U^*(2m)$.

Jonah Gaster: *A skinning map that is not one-to-one, and has a critical point*

Tuesday 7/31 – 3:50pm

Abstract: Thurston introduced the 'skinning map' of certain 3-manifolds, a holomorphic map from the Teichmüller space of a Riemann surface to itself. Apart from a simple class of examples, and recent numerical work (in preparation) of Dumas and Kent, explicit examples of skinning maps remain unexplored. In fact, it is consistent with the current literature to ask if skinning maps are always diffeomorphisms onto their images. We present a negative answer: There exists a hyperbolic structure on the genus-2 handlebody, with two rank-1 cusps, whose skinning map is not one-to-one, and has a critical point.

Olivier Guichard: *Maximal representations* (mini-course)

Tuesday 7/31 – 1:45pm; Thursday 8/2 – 9:00am; Friday 8/3 – 1:45pm

Abstract: Maximal representations are a family of representations of surface groups (i.e. $\pi_1(\Sigma_g)$) into the symplectic groups $\mathrm{Sp}(2n,\mathbf{R})$ (or, more generally, into Lie groups of Hermitian type). By many aspects, maximal representations generalize uniformization's representations and their moduli spaces generalizes the Teichmüller space. We will explain some of those aspects: discreteness and faithfulness of maximal representations, "limit circles", etc.

A tentative plan for the 3 lectures is the following:

1) maximal representations into $\mathrm{SL}(2,\mathbf{R})$, $\mathrm{PSL}(2,\mathbf{R})$ and... $\mathrm{Homeo}^+(S^1)$.
2) maximal representations into $\mathrm{Sp}(2n,\mathbf{R})$; Toledo number; geometry of $\mathrm{Sp}(2n,\mathbf{R})$ and of the Lagrangian variety.
3) equivariant maps into the Lagrangian variety; positivity; discreteness and faithfulness of maximal representations.

Thomas Haettel: *Visual limits of maximal flats in symmetric spaces and Euclidean buildings*

Tuesday 7/24 – 3:50pm

Abstract: We study the space of maximal flats of symmetric spaces of non-compact type or of Euclidean buildings: we define a geometric compactification by looking at the visual limits of a diverging sequence of flats. We completely determine the possible degenerations of flats when X is of rank 1, associated to a classical group of rank 2 or to PGL(4). In particular, we exhibit remarkable behaviours of visual limits of maximal flats in various symmetric spaces of small rank and surprising algebraic restrictions that occur.

Darren Long: *Hyperbolic manifolds: Some other families of representations* (mini-course)

Monday 7/30 – 3:15pm; Tuesday 7/31 – 9:00am; Thursday 8/2 – 3:15pm

Abstract: It's been explained elsewhere how one can recover geometric information from representations into certain Lie groups, for example $\mathrm{SL}(2,\mathbb{C})$. We'll discuss representations into other groups, (for example finite groups and $\mathrm{SL}(n,\mathbb{R})$) and give examples of some applications.

Brice Loustau: *Hyperkahler metrics on complex projective structures*

Friday 8/3 – 3:15pm

Abstract: I will discuss complex symplectic and hyperkahler structures on the moduli space of complex projective structures on a closed hyperbolic surface.

Kathryn Mann: *Two (or more) reasons to care about 3-manifold representations in* $\mathrm{PSL}(2,\mathbb{R})$

Friday 7/27 – 3:50pm

Abstract: You may be familiar with representations of 3-manifold groups into $\mathrm{PSL}(2,\mathbb{C})$ – for example, those defining a hyperbolic structure – but in this talk I'll convince you that it is also an interesting (and fundamental) question to study representations of hyperbolic 3-manifold groups into $\mathrm{PSL}(2,\mathbb{R})$. These representations are related to arithmetic/number theory questions (via trace fields), to foliations with transverse structures, to dynamics of mapping class elements on character varieties of surface groups, and more. In this short talk, I will try to paint a rough picture of what is known and what information such representations give, as well as bring up what I think are some interesting questions.

Karin Melnick: *Introduction to (G,X)-structures* (mini-course)

Monday 7/30 – 1:45pm; Wednesday 8/1 – 9:00am; Thursday 8/2 – 1:45pm

Abstract: A manifold with a (G,X)-structure is one locally modeled on a homogeneous space X. Such a structure endows the manifold with some geometry; this point of view was proposed by Felix Klein in his 1872 Erlangen Program. I will present the definition of a (G,X)-structure, together with the associated developing map and holonomy. There will be plenty of examples of (G,X)-structures of different kinds. I will talk about some classification results and open conjectures. In the third lecture, I'll define Cartan geometries, which are manifolds infinitesimally modeled on homogeneous spaces, and will show that in this setting the curvature is the obstruction to admitting a (G,X)-structure for the corresponding X.

Babak Modami: *Prescribing the behavior of Weil-Petersson geodesics*

Tuesday 7/24 – 4:30pm

Abstract: Brock, Masur and Minsky introduced a notion of ending lamination for Weil-Petersson (WP) geodesic rays analogous to the vertical foliations of Teichmüller geodesic rays. There are subsurface coefficients assigned to any pair of ending laminations analogue to continued fractions. I will show for a class of WP geodesic segments (geodesics with narrow ending data) the subsurface coefficients roughly determine the length-functions and twist parameters along the geodesic. Using which we would be able to give examples of closed WP geodesics staying outside any compact part of moduli space, as well as diverging WP rays with any rate of divergence and minimal-filling ending laminations.

Andrew Sanders: *Minimal surfaces in hyperbolic 3-manifolds*

Friday 7/27 – 3:15pm

Abstract: An almost-Fuchsian hyperbolic 3-manifold is a quasi-Fuchsian 3-manifold which contains a closed, incompressible minimal surface whose positive principal curvature is bounded above by one. We will discuss a few special features of these manifolds and describe an interesting circle action on the deformation space of almost-Fuchsian manifolds.

Nicolas Tholozan: (G, X)*-structures: an introduction to the problem of completeness*

Tuesday 7/31 – 3:15pm

Abstract: A (G, X)-structure on a manifold M is said to be complete if it identifies M to a quotient of $\tilde{X}$ by a discrete subgroup of G. A very general question is, given a G-homogeneous space X, is any (G, X)-structure on a compact manifold complete? If X is endowed with a G-invariant Riemannian metric, a positive answer naturally comes from the Riemann-Roch theorem. But in many cases this question is open (for instance if this G-invariant metric is only pseudo-Riemannian).

Carrière and Klingler solved the case where X is endowed with a Lorentzian metric of constant curvature. I would like to show how their ideas could generalize to different cases, and present some steps I've done toward a similar result in the case where X is a rank 1 Lie group and G is $X \times X$ acting by left and right translations.

Tian Yang: *A deformation of Penner's simplicial coordinate*

Friday 8/3 – 3:50pm

Abstract: Decorated Teichmüller space of a punctured surface was introduced by R. Penner as a fiber bundle over the Teichmüller space of complete hyperbolic metrics with finite area. To give a cell decomposition of this space that is invariant under the action of the mapping class group, Penner defined the simplicial coordinate Ψ in which the cells can be easily described. The main result that I will be talking about is to find a one-parameter family of coordinates Ψ_h that deforms Ψ. The decorated Teichmüller space turns out to be explicit convex open polytopes in these deformed coordinates. As an application, Bowditch-Epstein and Penner's cell decomposition is reproduced. Some further questions will be mentioned at the end of the talk.

Alfonso Zamora: *A GIT interpretation of the Harder-Narasimhan filtration*

Tuesday 7/31 – 4:30pm

Abstract: In a moduli problem where we use Geometric Invariant Theory to take the quotient to get a moduli space, an unstable object gives a GIT-unstable point in certain Quot scheme. To a GIT-unstable point, Kempf associates a "maximally destabilizing" 1-parameter subgroup, and this induces a filtration of the object. We show that this filtration coincides with the Harder-Narasimhan filtration for torsion free sheaves over smooth projective varieties, Higgs bundles and holomorphic pairs. We discuss the possibility of extending our method to construct Harder-Narasimhan filtrations in cases where it is not known. This is joint work with Toms L. Gmez and Ignacio Sols.