GEAR Retreat – Abstracts

August 6 – August 10, 2012

University of Illinois at Urbana-Champaign

All talks will be in 314 Altgeld Hall

David Aulicino: *Teichmüller discs with completely degenerate Kontsevich-Zorich spectrum*

Thursday 8/9 – 2:40pm

Abstract: We consider the Lyapunov exponents of the Kontsevich-Zorich cocycle over the moduli space of Abelian differentials with respect to $SL(2, \mathbb{R})$-invariant ergodic probability measures. Eskin-Kontsevich-Zorich asked for a classification of all such measures whose Lyapunov spectrum is completely degenerate, i.e. $1 = \lambda_1 > \lambda_2 = \cdots = \lambda_g = 0$. We approach the problem by studying $SL(2, \mathbb{R})$-invariant sets called Teichmüller discs, and prove that their closures cannot support a measure with completely degenerate spectrum for $g = 2$ and $g \geq 13$. In genus three and four, we prove that the known examples are the only ones. Finally, if there are no square-tiled surfaces in genus five whose orbit supports such a measure, then there are no such measures for $g \geq 5$. Time permitting we discuss partial results (with Vincent Delecroix) toward showing that no such surfaces in genus five exist.

Ian Biringer: *Extension into handlebodies and limits of Kleinian groups*

Wednesday 8/8 – 2:00pm

Abstract: We will discuss work with Johnson-Minsky and Lecuire concerning the extension of homeomorphisms of the boundary of a handlebody into its interior. Although the motivation is topological, our techniques come from 3-dimensional hyperbolic geometry.

Dick Canary: *Deformation space of hyperbolic 3-manifolds*

Wednesday 8/8 – 9:30am

Abstract: We will survey what is known about deformation spaces of hyperbolic 3-manifolds, and then focus on some of the remaining questions about their geometry and topology.

Daryl Cooper: *Geometric Structures*

Friday 8/10 – 9:30am

Abstract: This will survey some results on geometric structures on manifolds.

Jeff Danciger: *Complete Lorentz spacetimes in dimension three, II*

Friday 8/10 – 3:20pm

Abstract: A complete flat Lorentz spacetime in dimension three is a quotient of $\mathbb{R}^3$ by a discrete group acting properly by affine $O(2, 1)$ transformations. In the interesting cases, the linear part of the action is the fundamental group of a noncompact hyperbolic surface, which we assume to be convex cocompact, and the translational part of the action corresponds to an infinitesimal deformation of this surface group. Following work of Guéritaud and Kassel in the negatively curved (AdS) case, we prove that such an action is proper if and only if this group-level deformation is realized by a deformation of the surface that everywhere contracts distances at a uniform rate. We then give two applications. 1) Tameness: A complete flat spacetime is diffeomorphic to a handle body. 2) Geometric degeneration: A complete flat spacetime is the limit of collapsing negatively curved (AdS) spacetimes. This is joint work with François Guéritaud and Fanny Kassel.

Spencer Dowdall: *Kleinian convex cocompact subgroups of mapping class groups*

Friday 8/10 – 2:00pm

Abstract: Convex cocompact subgroups of mapping class groups, as introduced by Farb and Mosher, are subgroups whose action on Teichmüller space is analogous to that of convex cocompact Kleinian groups

acting on $\mathbb{H}^3$. In this talk I will describe a setting in which there is a concrete connection between these two notions of convex cocompactness: Because of the Birman exact sequence, fibered hyperbolic 3-manifold groups give many ways to realize Kleinian groups as subgroups of mapping class groups. Expanding on earlier work of Kent–Leininger–Schleimer, I will explain a result that certain subgroups of these 3-manifold groups which are convex cocompact in the Kleinian sense are also convex cocompact as subgroups of the mapping class group. This is joint work with Richard Kent and Christopher Leininger.

Guillaume Dreyer: *Geometric properties of Anosov representations*

Monday 8/6 – 2:00pm

Abstract: Let S be a connected, closed, oriented surface of negative Euler characteristic. We consider the $\mathrm{PSL}_n(\mathbb{R})$–character variety $\mathrm{Rep}_{\mathrm{PSL}_n(\mathbb{R})}(S)$. An interesting connected component of the latter space is the Hitchin space $\mathcal{H}(S)$: it contains a copy of the Teichmüller space $\mathcal{T}(S)$, and hence is regarded as the higher rank Teichmüller space in the case of $\mathrm{PSL}_n(\mathbb{R})$. In order to study the elements in the space $\mathcal{H}(S)$, F. Labourie introduced the notion of Anosov representation. In particular, he proved that every Anosov representation is discrete and injective, some properties already shared by Teichmüller representations. The purpose of this talk is to extend to Anosov representations some classic tools from hyperbolic geometry designed to study Teichmüller representations: we generalize Thurston's length function and cataclysm deformation, and we analyse how they relate to each other. We shall then discuss how these techniques provide crucial information about a new system of coordinates on the Hitchin space $\mathcal{H}(S)$.

David Dumas: *Osculation in complex projective and hyperbolic structures*

Friday 8/10 – 11:00am

Abstract: We will discuss how complex-analytic methods from Teichmüller theory apply naturally to the study of various geometric structures on surfaces, with the concept of osculation being a central theme in these connections. While complex projective structures will be the main example in this discussion, we will touch on some other settings where these methods apply.

Nathan Dunfield: *Surfaces in finite covers of 3-manifolds: the virtual Haken conjecture*

Wednesday 8/8 – 11:00am

Abstract: I will sketch the recent breakthrough result of Agol that every irreducible 3-manifold with infinite fundamental group has a finite cover which is Haken, that is, contains an embedded incompressible surface. This work builds on critical contributions of Wise, Kahn-Markovic, and Agol-Groves-Manning, which will also be discussed.

Oscar Garcia-Prada: *Higgs bundles and representations of surface groups*

Tuesday 8/7 – 11:00am

Abstract: We will explain how Higgs bundles over a Riemann surface can be used to study representations of the fundamental group of the surface in a real semisimple Lie group. Special attention will be given to the case in which the group is split or Hermitian. The case of a surface with punctures will also be discussed.

Alberto Garcia-Raboso: *A non-abelian Hodge theorem for twisted vector bundles*

Tuesday 8/7 – 2:00pm

Abstract: I will discuss a generalization of the non-abelian Hodge theorem of Corlette and Simpson. This correspondence is built up in two stages. In the first one, abelian Hodge theory relates G_m-gerbes with flat connection over a smooth complex projective variety X to Higgs G_m-gerbes over X. The category of vector bundles on the flat gerbe equipped with a connection compatible with that of the gerbe is then equivalent to the category of appropriately defined Higgs bundles over the corresponding Higgs gerbe. An interesting case of this correspondence is that of locally free twisted D-modules on X in the sense of Beilinson and Bernstein.

Fanny Kassel: *Complete Lorentz spacetimes in dimension three, I*

Friday 8/10 – 2:40pm

Abstract: A complete flat Lorentz spacetime in dimension three is a quotient of $\mathbb{R}^3$ by a discrete group acting properly by affine $O(2,1)$ transformations. In the interesting cases, the linear part of the action is the fundamental group of a noncompact hyperbolic surface, which we assume to be convex cocompact, and the translational part of the action corresponds to an infinitesimal deformation of this surface group. Following work of Guéritaud and Kassel in the negatively curved (AdS) case, we prove that such an action is proper if and only if this group-level deformation is realized by a deformation of the surface that everywhere contracts distances at a uniform rate. We then give two applications. 1) Tameness: A complete flat spacetime is diffeomorphic to a handle body. 2) Geometric degeneration: A complete flat spacetime is the limit of collapsing negatively curved (AdS) spacetimes. This is joint work with Jeff Danciger and François Guéritaud.

Francois Labourie: *Convex Anosov representations, Intersection and the Pressure Metric*

Monday 8/6 – 9:30am

Abstract: In this talk, I will discuss convex Anosov representations which form an easily defined, although fairly general, case of Anosov representations. I will then explain how the the thermodynamic formalism work in this context and in particular prove analyticity of entropy. Using the concept of intersection and the pressure metric, I will define a Riemannian metric on the deformation space of convex Anosov representations. This is a joint work with Martin Bridgeman, Dick Canary and Andres Sambarino.

Michelle Lee: *Dynamics on the* $\mathrm{PSL}(2,\mathbb{C})$*-character variety of certain hyperbolic* 3*-manifolds*

Thursday 8/9 – 2:00pm

Abstract: The $\mathrm{PSL}(2,\mathbb{C})$-character variety of a hyperbolic 3-manifold M is essentially the set of homomorphisms of the fundamental group of M into $\mathrm{PSL}(2,\mathbb{C})$, up to conjugacy. We will discuss the action of the group of outer automorphisms of the fundamental group of M on this space. In particular, we will discuss how one can find domains of discontinuity for the action.

Sara Maloni: *Plumbing and slices of Quasifuchsian space*

Wednesday 8/8 – 3:20pm

Abstract: Given a surface S, Kra's plumbing construction endows S with a projective structure for which the associated holonomy representation depends on some complex 'plumbing parameters'. After reviewing the definition of the Dehn-Thurston coordinates for simple closed curves, of the Maskit embedding, of the pleating rays and of the plumbing construction, we will describe some new results involving these topics. Time permitting, we will define other slices of the Quasifuchsian space $\mathrm{QF}(S)$, some properties satisfied by these slices and we will show some pictures.

Howard Masur: *Weil-Petersson metric on Teichmüller space*

Thursday 8/9 – 11:00am

Abstract: The Weil-Petersson metric is an important metric on moduli space which has connections to hyperbolic geometry. I plan to survey some recent developments and perhaps indicate some open problems.

Andy Neitzke: *Spectral networks and character varieties*

Tuesday 8/7 – 2:40pm

Abstract: A spectral network is a network of paths drawn on a punctured surface, satisfying certain combinatorial rules. I will explain how spectral networks induce coordinate systems on (finite covers of) character varieties for the group $\mathrm{GL}(K)$. Conjecturally, these coordinate systems are those in the cluster atlas for the character variety, as described by Fock-Goncharov. Spectral networks are also a key ingredient in a new description of the hyperkahler structure on the character variety. This is joint work with Davide Gaiotto and Greg Moore.

Tony Pantev: *Higgs bundles and representations of fundamental groups*

Tuesday 8/7 – 9:30am

Abstract: I will review the notion of a Higgs bundle on a Riemann surface and will discuss the non-abelian Hodge theorem which relates representations of fundamental groups to Higgs bundles. I will explain how the non-abelian Hodge dictionary can be used to convert difficult existence questions for special representations to projective geometry. I will present samples of standard construction techniques, including the spectral cover construction and Katz's middle convolution algorithm. I will also explain how this formalism can be applied to construct rigid representations, representations with prescribed arithmetic properties, and representations of geometric origin.

Robert Rubio: *Higgs bundles and Hermitian symmetric spaces*

Tuesday 8/7 – 3:20pm

Abstract: We study the moduli space of polystable G-Higgs bundles for noncompact real Lie groups G of Hermitian type. First, we define the Toledo character and use it to define the Toledo invariant, for which a Milnor-Wood type inequality is proved. Then, for the maximal value of the Toledo invariant, we state a Cayley correspondence for groups of so-called tube type and point out a rigidity theorem for groups of so-called non-tube type. The proofs of these results are based on the Jordan algebra structure related to the tangent space of the Hermitian symmetric space given by G and are independent of the classification theorem of Lie groups. (Joint work with O. Biquard and . Garca-Prada.)

Andres Sambarino: *On the convex core of a representation in the Hitchin component*

Monday 8/6 – 2:40pm

Abstract: The purpose of the talk is to give a description of the convex core, together with the action of the Weyl chamber flow, of a representation on the Hitchin component.

John Smillie: *Translation surfaces and Teichmüller dynamics*

Thursday 8/9 – 9:30am

Abstract: This talk will be an introduction to translation surfaces, their moduli spaces and certain natural dynamical systems defined on their moduli spaces. I will describe some motivating examples, some known results and some open questions.

Tobias Strubel: *Cross ratios for maximal representations*

Monday 8/6 – 3:20pm

Abstract: Strict cross ratios associated with Hitchin representations have been introduced by Labourie. In my talk I will present a construction of a strict cross ratio for every maximal representation of a surface group into $\mathrm{Sp}(2n, \mathbb{R})$, which can be generalized to all maximal representations into Hermitian Lie groups of tube type. This is joint work with Tobias Hartnick.

Anastasiia Tsvietkova: *Hyperbolic structures from link diagrams*

Wednesday 8/8 – 2:40pm

Abstract: W. Thurston demonstrated that every link in S^3 is a torus link, a satellite link or a hyperbolic link and these three categories are mutually exclusive. It also follows from work of W. Menasco that an alternating link represented by a prime diagram is either hyperbolic or a $(2; n)$-torus link. The talk will introduce an alternative method for computing the hyperbolic structure of the complement of a hyperbolic link. It allows computing the structure directly from the link diagram. Some of its consequences will be discussed, including a surprising rigidity property of certain tangles, and the formulas that allow one to calculate the exact hyperbolic volume, as well as complex volume, of hyperbolic 2-bridge links. This is joint work with M. Thistlethwaite.

Anna Wienhard: *Higher Teichmüller spaces and Anosov representations*

Monday 8/6 – 11:00am

Abstract: I will put several results concerning special subsets of representation varieties (e.g. Hitchin representations, positive representations or maximal representations, as well as Anosov representations) into context, emphasizing aspects which let them appear as suitable generalizations of Teichmüller spaces, quasi-Fuchsian representations or convex cocompact subgroups for target groups of higher rank (e.g. $\mathrm{SL}(n, \mathbb{R})$ instead of $\mathrm{SL}(2, \mathbb{R})$).

Alex Wright: $\mathrm{SL}(2, \mathbb{R})$ *orbit closures in the moduli of translation surfaces*

Thursday 8/9 – 3:20pm

Abstract: We will present some new results on $\mathrm{SL}(2, \mathbb{R})$ orbit closures in the moduli of translation surfaces.