GEAR Second Network Retreat, University of Maryland
March 17–21, 2014
Abstracts

Jørgen Anderson, Center for Quantum Mathematics, Danish Institute for Advanced Study
"Quantum representations of mapping class groups via geometric quantization of moduli spaces"

We will describe how the Reshetikhin-Turaev quantum representations of mapping class groups are constructed by applying geometric quantization to the moduli space of flat SU(n)-connections on the surface. We will further show how we can generalise this to also apply geometric quantization to the moduli space of flat SL(n,C)-connections to obtain new infinite dimensional quantum representations of mapping class groups corresponding to quantum Chern-Simons theory for the gauge group SL(n,C).

Michelle Bucher, Université de Genève
"Volume and characteristic numbers of representations of hyperbolic manifolds"

Let G be a lattice in SO(n,1) and let h:G --> SO(n,1) be any representation. For cocompact lattices, the volume of a representation is an invariant whose maximal and rigidity properties have been studied extensively. We will show how to define the volume of a representation in the non cocompact case (a different definition by Francaviglia and Klaff also exists). In particular, we establish a rigidity result for maximal representations, recovering Mostow rigidity for hyperbolic manifolds.
In the cocompact case, the set of values for the volume of a representation is discrete. In even dimension, this follows from the fact that the volume form is an Euler class. In odd dimension, this was proven by Besson, Courtois and Gallot. The situation changes in the noncocompact case and for example the discreteness of the set of value is not valid anymore in dimension 2 and 3. We prove that in even dimension greater or equal to 4, the set of value of the volume of a representation is, up to a universal constant, an integer.

Moira Chas, Stony Brook University
"Computer driven theorems and questions in geometry"

Given an orientable surface S with negative Euler characteristic, a minimal set of generators of the fundamental group of S, and a hyperbolic metric on S, each free homotopy class C of closed oriented curves on S determines three numbers: the word length (that is, the minimal number of generators and inverses necessary to express C as a cyclically reduced word), the minimal self-intersection, and the geometric length. On the other hand, the set of free homotopy classes of closed directed curves on S (as a set) is the basis of a Lie algebra structure (discovered by Goldman). This Lie algebra is closely related to the intersection structure of curves on S. These three numbers, as well as the Goldman Lie bracket of two classes, can be explicitly computed (or approximated) by the help of computer. These computations lead us find counterexamples to existing conjectures and to establish new conjectures.

Thomas Church, Stanford University
"A survey of representation stability"

I will give a gentle survey of the theory of representation stability, viewed through the lens of its applications. These applications include: homological stability for configuration spaces of manifolds;

understanding the stable (and unstable) homology of arithmetic lattices; Hecke eigen classes in stable mod-p cohomology; uniform generators for congruence subgroups and "congruence" subgroups; and distributional stability for random square free polynomials over finite fields.

Sorin Dumitrescu, Université Côte d'Azur
“Quasihomogeneous analytic affine connections on surfaces”

In this joint work with Adolfo Guillot, we classify real analytic torsion free affine connections on compact oriented surfaces which are quasihomogeneous, in the sense that they are locally homogeneous on nontrivial open sets, without being locally homogeneous on all of the surface. The proof relies on a local result which classifies quasihomogeneous germs at the origin of real analytic torsion free connections in dimension two. I will also explain our motivations which come from Gromov's open-dense orbit theorem.

Elisha Fabel, Institut de Mathématiques de Jussieu
“Combinatorics of 3-manifolds, geometric structures, and representations into PGL(3,C)”

I will explain how to obtain information about the representation space of the fundamental group of a manifold of dimension 3 into PGL(3,C) from a triangulation of that manifold. The combinatorics of the triangulation is coded by flags (points and lines containing them) in projective space of complex dimension 2. The geometric structures associated to these representations include CR structures and real flag structures.

Francois Guéritaud, University of Lille
“Arc complex and fundamental domains for Margulis spacetimes”

Lorentzian manifolds with constant nonpositive curvature are spaces modelled on the group PGL(2,R) or its Lie algebra. They can be understood in terms of Fuchsian representations of the fundamental group of a surface S. I will explain how such a manifold determines a point of the arc complex (a combinatorial object associated to S) and how to associate to this point a natural fundamental domain bounded by "crooked planes." This is joint work with J. Danciger and F. Kassel.

Tobias Hartnick, Karlsruhe Institute of Technology
“Bounded cohomology of Lie groups and its uses in higher Teichmüller theory”

We explain how the second bounded cohomology of Lie groups can be used in order to establish properties of higher Teichmüller representations such as faithfulness and discreteness. We then survey various results on higher degree bounded cohomology of Lie groups and point out some major differences between bounded cohomology in degree 2 and 3 and higher degree bounded cohomology. The first part is based on joint work with Gabi Ben Simon, Marc Burger, Alessandra Iozzi and Anna Wienhard, whereas the second part is based on joint work with Andreas Ott.

Alessandra Iozzi, Eidgenössische Technische Hochschule Zürich
“Rotation numbers, old and new”

Rotation numbers classify orientation preserving homeomorphisms of the circle (hence actions of the integers) up to semiconjugacy. After recalling the classical theory, we will show how one can generalize the notion of rotation number both to actions of groups larger than the integers and to actions on manifolds more complicated than the circle. As an application, we will illustrate some rigidity results.

Michael Kapovich, University of California, Davis
“On representation varieties of 3-manifold groups”

We prove universality theorems ("Murphy's Laws") for representation varieties of fundamental groups of closed 3-dimensional manifolds. We show that germs of SL(2;C)-representation schemes of such groups are essentially the same as germs of schemes of finite type over Q. This is a joint work with John Millson.

Steve Kerckhoff, Stanford University
“Transitional geometry”

Transitional geometry is the study of transitions between sub-geometries of a larger geometry, particularly projective geometry. We'll explore this idea through some low-dimensional examples.

Chris Leininger, Rice University
“Geometry and dynamics of splittings of free-by-cyclic groups”

Thurston proved that the mapping torus M of a pseudo-Anosov homeomorphism f of a surface S admits a hyperbolic metric. However, it also admits a singular solv metric which more closely relates to the geometry and dynamics of f. Thurston and Fried proved that the different ways in which M can be expressed as a mapping torus is organized by the primitive integral points in a finite family of cones in the first cohomology of M, and that the dynamics are governed by a convex function on each of these cones. For each cone, McMullen defined his Teichmueller polynomial which determines the convex function and proves the real analyticity of it. Furthermore, his analysis produces generalizations of the singular solv metrics which vary continuously over the entire cone and interpolate between the singular solv metrics from the integral points.

Bestvina and Handel's train-track maps of graphs provide an analogue for pseudo-Anosov homeomorphisms in the realm of automorphisms of free groups. In this talk, I will describe on-going joint work with S. Dowdall and I. Kapovich which develops the analogous structure on (a variant of) the mapping torus of the train-track map. After describing the set-up coming from our earlier work, I will focus attention on the construction of an open cone, convex real-analytic function, a polynomial invariant, and the continuously varying geometric structures which tie together the geometry, topology, and dynamics.

John Loftin, Rutgers University
“Convex real projective structures and cubic differentials”

Given a cubic differential U on a Riemann surface with background conformal metric mu, conformally perturb the metric to be e^umu$, where u satisfies the equation

$$Delta_mu u pm 16|U|^2_mu e^{-2u} pm 2e^u - 2kappa_mu=0.$$

Then e^umu is a natural Riemannian metric on a surface (either an affine sphere or a minimal Lagrangian surface in a Hermitian symmetric space, depending on the signs), and the structure equations of the surface can be integrated once the metric e^umu and cubic differential U are in place. The structure equations give rise to a flat connection on a principal bundle with structure group a real form G of $SL(3,mathbb C)$, and thus to a representation of the fundamental group into G. We will discuss the cases of a hyperbolic affine sphere (signs $+,-$) and minimal Lagrangian surface in $mathbb{CH}^2$ (signs $-,-$) in more detail, and describe the geometry of the surfaces and the corresponding representations.

Ignasi Mandet Riera, University of Barcelona
"The Cayley transform for symplectic Higgs bundles"

The Cayley transform for symplectic Higgs bundles transforms symplectic Higgs bundles of extremal Toledo invariant into K^2-twisted Higgs bundles for the general linear group. This correspondence is compatible with families and preserves the respective notions of (semi)stability, so it induces an isomorphism of moduli spaces. We will discuss the algebraic side of the construction, which is well established, and the analytic side (i.e., its relation to the Hitchin-Kobayashi correspondence), which is largely conjectural.

Walter Neumann, Barnard College
"Representations and the Bloch group"

This will be an overview of results and open questions regarding the Bloch group and invariants of manifolds and representations in the Bloch group.

Dragomir Saric, Queens College, CUNY
"Thurston's boundary for Teichmuller spaces of infinite surfaces"

We introduce Thurston's boundary for Teichmuller spaces of infinite surfaces using geodesic currents and the length spectrum. Bonahon used geodesic currents to introduce Thurston's boundary to Teichmuller spaces of closed surfaces. We used geodesic currents for arbitrary hyperbolic surfaces in order to introduce Thurston's boundary to their Teichmuller spaces which turned out to be the space of projective bounded measured laminations on the base surface. Our first contribution is to simplify this description. Moreover, we define Thurston's boundary using the length spectrum for infinite surfaces. It turns out that the length spectrum Thurston's boundary always contains the projective bounded measured laminations but it is larger in general.

Jean-Marc Schlenker, University of Luxembourg
"Polyhedra inscribed in a hyperboloid and anti-de Sitter geometry"

Consider a graph G embedded in the 2-dimensional sphere. When is G the 1-skeleton of a convex polyhedron inscribed in a 1-sheeted hyperboloid? We prove that this happens if and only if G is the 1-skeleton of a polyhedron inscribed in the sphere and it has a Hamiltonian cycle. This is a by-product of a description of the possible dihedral angles and induced metrics on ideal polyhedra in the 3-dimensional anti-de Sitter space. Joint work with Jeff Danciger and Sara Maloni.

Amie Wilkinson, University of Chicago
"Dynamics and incomplete geometry"

Abstract not available

Michael Wolf, Rice University
"Polynomial Pick forms for affine spheres and real projective polygons"

(Joint work with David Dumas.) Discrete surface group representations into PSL(3, R) correspond geometrically to convex real projective structures on surfaces; in turn, these may be studied by considering the affine spheres which project to the convex hull of their universal covers. As a sequence of convex projective structures leaves all compacta in its deformation space, a subclass of the limits is described by polynomial cubic differentials on affine spheres which are conformally the complex plane. We show that those particular affine spheres project to polygons; along the way, a strong estimate on asymptotics is found, which translates to a version of the Stokes data. Relying somewhat on the constructions in the talk by John Loftin, we begin by describing the basic objects and context and conclude with a sketch of some of the useful technique.