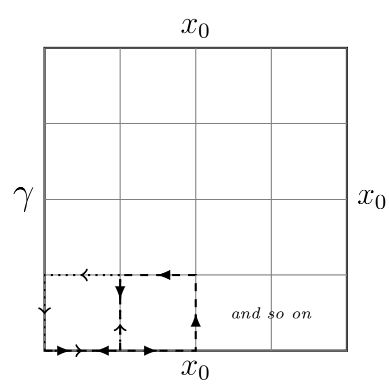
GEAR 2012: Introduction to (G, X)-structures
Problem Set 1
K. Melnick

1. Let X be a manifold, and suppose Γ acts on X properly discontinously. Show that the quotient space $\Gamma\backslash X$ is Hausdorff.

2. Assume $g \in \mathrm{GL}(n, \mathbf{R})$ is semisimple (diagonalizable over $\mathbf{C}$). Show that $\langle g \rangle$ acts freely and properly discontinuously on $\mathbf{R}^n \backslash \{0\}$ if and only if the eigenvalues $\lambda_1, \ldots, \lambda_n$ satisfy $|\lambda_i| > 1 \ \forall i$ or $|\lambda_i| < 1 \ \forall i$.

3. Show that if $\gamma : [0, 1] \to M$ is a null homotopic loop, say at x_0, then development along γ has the same value at $t = 1$ as at $t = 0$.

Hint:



4. What are all similarity structures on S^1? Which are complete?

5. Show that an affine manifold M is geodesically complete if and only if it is complete as a (G, X)-manifold.