GEAR problems

1. Prove the assertion that residual finiteness is equivalent to solving the geometric mapping problem that was described.

Solve this mapping problem for the wedge of two circles, i.e. use this geometric approach to prove the free group of rank 2 is residually finite.

2. Prove that if $H \leq G$ of finite index and H is subgroup separable, then so is G.

3. Construct a non-separable subgroup in a free group of rank 2.

4. Show there must be a non-congruence subgroup in $SL(2, \mathbf{Z})$. (Can you exhibit one?)*

5. Suppose that G is residually finite and that A is a maximal abelian subgroup of G. (i.e. A is abelian and the maximal such subgroup of G). Prove that A is separable in G.

Deduce that in a finite volume hyperbolic 3-manifold group (i) the cyclic subgroup corresponding to a primitive hyperbolic element is separable and (ii) a cusp subgroup is separable.

6. A subgroup H of a finite volume hyperbolic 3-manifold group G is *totally geodesic* if G can be conjugated inside $PSL(2, \mathbf{C})$ so that H represents into $PSL(2, \mathbf{R})$. Prove that maximal totally geodesic subgroups are separable.

7. (Requires some knowledge of Coxeter groups and their properties) Prove that the special subgroups of a Coxeter group are separable.