

QUESTIONS FOR GEAR MEMBERS

The following questions were submitted anonymously online for the Everything You Want to Know (EYWTK) Session at the 2012 GEAR Retreat in Urbana-Champaign. The session was moderated by Steve Kerckhoff and Mike Wolf.

- (1) Can we have a kind of Higgs bundles over singular curves ?
- (2) It was mentioned (in Tony Pantev's talk) that Higgs bundles are easy to construct, and they are abundant in nature. What would be some elementary examples to see and more generally, what are some references where one could read about it? For example, what are some algebro-geometric constructions of such objects?
- (3) An algebraic geometer would get very upset if I just referred to a "moduli space" without saying whether it is coarse, fine, or even worse, a stack. And I would need to worry about what constitutes a flat family. Otherwise they would derisively say that one is just talking about a "parameter space". But algebraic geometers do extract a lot of information about singularities and compactifications from such considerations. Are these considerations as important for the "moduli spaces / parameter spaces" of Higgs bundles or representations? Are there differential geometric equivalents or analogues of flat families, particularly near singularities?
- (4) Does 'higher' Teich theory have any significant impact on 'lower' Teich/moduli space theory in that it provides new insight? Or, should the 'higher' theory be regarded as a separate theory entirely, a theory about representations of surface groups into various Lie Groups of higher dimensions than $PSL(2, \mathbb{R})$. A theory having little to do with complex analysis, or reps of surface and 3-manifold groups into $PSL(2, \mathbb{C})$.
- (5) Which orientable 3-manifolds can contain an embedded projective plane or Klein bottle ? (RP^3 is one example)
- (6) Is there any kind of refined K-theory for Higgs Bundles ?
- (7) When do we have more than one geometric structures on a 3-manifolds (assuming it's geometric) ?
- (8) Can hyperbolic 3-manifolds admit non-boundary parallel incompressible annuli? Why ?
- (9) Labourie tells us that the boundary maps which plays a role in Anosov representations (a) exist, (b) are continuous, and (c) are positive. Do we know anything concrete about how these functions look? How should we think about them?
- (10) Do surface group representations into any non-reductive groups (maybe affine group, Poicare group) play any special role? Show anything interesting?
- (11) What exactly is the relation between surface groups and 3-manifold topology?

- (12) What is the role of non-orientable surfaces (or more generally Kleinian surfaces) in 3-manifold topology?
- (13) There are several combinatorial structures associated to quadratic differentials—train tracks, Interval Exchange Maps, etc. Do these have any meaning from the Higgs Bundle perspective?
- (14) How does the associated Higgs bundle change as we move along some trajectory (say a Teichmüller geodesic) in the space of quadratic differentials?