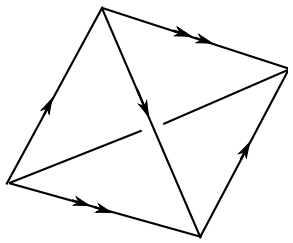
GEAR problems

1. Identify the front two faces of a tetrahedron as shown, leaving the back two faces free.



Which 3-manifold do you get? Which closed 3-manifolds are obtained by gluing two of these together?

2. Show that a closed hyperbolic 3-manifold is atoroidal. Show that all hyperbolic 3-manifolds are atoroidal.

3. Show that a hyperbolic 3-manifold is irreducible.

4. Triangulate the complement of the trefoil knot and solve the gluing equations. What happens?

5. The geometry of a closed geometric 3-manifold is unique, but the trefoil knot complement has two: $\mathbb{H}^2 \times \mathbb{R}$ and $\widetilde{SL}_2(\mathbb{R})$. Find these structures.