Geometry, Groups and Dynamics/GEAR Seminar
(held at the Illinois hub of GEAR)

12:00 pm, Tuesday, April 19, 2016, 243 Altgeld Hall

Anton Lukyanenko (University of Michigan)

Separated nets in the Heisenberg group

Abstract: A co-compact lattice is a standard example of a separated net, but other nets also arise in applications (most famously the quasi-crystals in chemistry), and one would like to know whether they are simply perturbations of lattices. In Euclidean space, a criterion of Laczkovich allows one to easily make nets that are not a bounded-distance perturbation of any lattice (not BD rectifiable), and an intricate construction due to McMullen and Burago-Kleiner provides a net that is not even bi-Lipschitz to any lattice (not BL rectifiable). We study nets and quasi-crystals in the Heisenberg group and more generally (rational) Carnot groups. Lattices in these groups are quite tame, and by a theorem of Malcev may even be viewed as the integer points in appropriate coordinates. We show that a generic net need not be well-behaved: in addition to nets that are not BD or BL rectifiable, there exist BD-rectifiable "exotic nets" that are neither coarsely dense nor uniformly discrete in Malcev coordinates. On the other hand, in applications, a natural construction of quasi-crystals yields easily-understood nets whose BD rectifiability is based on a certain Diophantine condition, showing that almost every Heisenberg quasi-crystal is a BD perturbation of a lattice.

Video (unavailable)