Geometry, Groups and Dynamics/GEAR Seminar
(held at the Illinois hub of GEAR)

12:00 pm, Thursday, November 30, 2017, 243 Altgeld Hall
Ilya Gekhtman (Yale University)
Green metric, Ancona inequalities and Martin boundary for relatively hyperbolic groups
Generalizing results of Ancona for hyperbolic groups, we prove that a random path between two points in a relatively hyperbolic group (e.g. a nonuniform lattice in hyperbolic space) has a uniformly high probability of passing any point on a word metric geodesic between them that is not inside a long subsegment close to a translate of a parabolic subgroup. We use this to relate three compactifications of the group: the Martin boundary associated with the random walk, the Bowditch boundary, associated to an action of the group on a proper hyperbolic space, and the Floyd boundary, obtained by a certain rescaling of the word metric. We demonstrate some dynamical consequences of these seemingly combinatorial results. For example, for a nonuniform lattice G in hyperbolic space H^n, we prove that the harmonic (exit) measure on the boundary associated to any finite support random walk on G is singular to the Lebesgue measure. Moreover, we construct a geodesic flow and G invariant measure on the unit tangent bundle of hyperbolic space projecting to a finite measure on $T^1 H^n / G$ whose geodesic current is equivalent to the square of the harmonic measure. The axes of random loxodromic elements in G equidistribute with respect to this measure. Analogous results hold for any geometrically finite subgroups of isometry groups of manifolds of pinched negative curvature, or even proper delta-hyperbolic metric spaces.

Video