Geometry, Groups and Dynamics/GEAR Seminar
(held at the Illinois hub of GEAR)

Thursday, April 9, 2015, 1:00 pm in 243 Altgeld Hall

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Clairaut's theorem and potential mechanics on metric spaces

Abstract: For surfaces of revolution Clairaut's theorem gives a first integral for geodesics: $r \cos\theta =$ constant, where r is the distance from the axis to the profile curve and θ is the angle the geodesic makes with the latitude circles. We have generalized this to warped products $W = B\times_fF$ of metric spaces: along any geodesic γ in W, $f^2v = b$ is constant, where v is the speed of the projection of γ to F. When B, F are Riemannian manifolds, the geodesic equations have a known form: $$ \gamma_B'' = c(1/f^3) {\rm grad} f, \qquad (f^2v)' =0,$$ where γ_B is the projection to B. This has the interpretation that γ_B is a trajectory of the potential function $U = c/2f^2$. The fact that the speed of γ is a constant $a = \sqrt{b/c}$ becomes the law of conservation of energy $u^2 + 2U = (b/c)^2$, where u is the speed of γ_B. Hence, for more general metric spaces B, Clairaut's theorem makes it reasonable to interpret the projections of geodesics from a warped product $B\times_fF$ to B as the trajectories of the potential function $U = 1/2f^2 :B \to {\bf R}$. Since we also have shown that these trajectories are independent of the choice of F, we can simply take F to be the line or a circle. Joint work with Stephanie Alexander.

Video