Junior GEAR Retreat
Curves, surfaces and hyperbolic 3-manifolds

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1 Problem Session

The following problems flesh out some of the examples and assertions made in the lecture. Some of them assume a bit of background on the classification of mapping classes.

1. Let M_f be the fibered manifold with monodromy $f : S \to S$. Explain why f being pseudo-Anosov is a necessary condition for M_f being hyperbolic.

2. With M_f as above, if $\tau : S \to S$ is a Dehn twist, describe $M_{f\tau^n}$ as a Dehn surgery on M_f.

3. Let $f : S \to S$ be pseudo-Anosov and let $\tau : S \to S$ be a Dehn twist. Show that, for n large enough, $f \circ \tau^n$ is pseudo-Anosov.

4. As a special case to the problem above, repeat with S a torus, f a hyperbolic matrix in $SL(2, \mathbb{Z})$, and $\tau = \left[\begin{smallmatrix} 1 & 1 \\ 0 & 1 \end{smallmatrix}\right]$.

5. Let $A \in PSL(2, \mathbb{C})$ be hyperbolic, with complex translation length $\lambda = \ell + i\theta$.

   If $\ell = \theta^2 << 1$, estimate the radius of the Margulis tube of A and show that its boundary is a torus with area bounded above.

6. Given a curve c in a closed hyperbolic surface S, let T be a triangulation of S containing c in its 1-skeleton with a single vertex that lies on c. How many triangles does T have?

   Apply to T a sequence of Dehn twists around c and show that the resulting sequence of triangulations tends to a lamination in an appropriate sense. How many leaves does the lamination have?

7. Let $S \subset N$ where N is hyperbolic and let α be a simple curve in S which is homotopic in N to a geodesic α^*. If $\gamma \subset S$ is another simple curve crossing α and τ is a Dehn-twist around α, describe the geodesic representatives of the curves $\tau^n\gamma$ (the answer may depend on properties of the embedding $S \subset N$).

8. Let $f : S \to N$ be a π_1-injective map. For a simple closed curve $\alpha \subset S$ let α^* be the geodesic representative in N of $f(\alpha)$. Let d_C denote distance in the complex of curves of S, and let d_N^ϵ denote "electrocuted" distance in N, meaning distance where each component of the ϵ-thin part is declared to have size 0. Show that $d_N^\epsilon(\alpha^*, \beta^*) < Ad_C(\alpha, \beta)$, where C depends on the genus of S.

2 Bibliography

For a brief expository account see [6]. Thurston's original paper [8] is still a rich rewarding read.

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3. J. Brock, R. Canary and Y. Minsky, *The classification of Kleinian surface groups II: the ending lamination conjecture*, Ann. of Math. (2) **176** (2012), 1–149.

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8. W. P. Thurston, *Hyperbolic Structures on 3-manifolds, II: Surface groups and 3-manifolds which fiber over the circle*, Arxiv math/9801045 (1986).