

Junior GEAR Retreat
*Geometry and physics of Higgs bundles and
branes*

Sergei Gukov

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1 Problem Session

1. Give an explicit description of the moduli space of Higgs bundles $\mathcal{M}_H(G, \Sigma)$ in each of the following examples:
 - a) $G = U(1)$ and $\Sigma =$ genus- g Riemann surface
 - b) $G = SU(2)$ and $\Sigma = T^2$
 - c) $G = SU(2)$ and $\Sigma = T^2 \setminus \{p\}$ with a “first order pole” at p
 - d) $G = SU(2)$ and $\Sigma = S^2 \setminus \{p_1, p_2, p_3, p_4\}$ with tame ramification at p_i , $i = 1, 2, 3, 4$
 - e*) $G = SU(2)$ and $\Sigma =$ genus-2 Riemann surface (without punctures)
2. In each of the above examples, propose a mirror manifold to $\mathcal{M}_H(G, \Sigma)$.
3. Pick a 3-manifold M_3 bounded by a 2-torus $T^2 = \partial M_3$ and describe the corresponding (A, B, A) brane \mathcal{B} in the Hitchin moduli space in part b) or Exercise 1. The brane \mathcal{B} is supported on $\mathcal{M}_{\text{flat}}(G_{\mathbb{C}}, M_3)$.
4. * Do the same for a genus-2 Riemann surface Σ : pick your favorite genus-2 handlebody M_3 and describe (A, B, A) brane \mathcal{B} supported on the moduli space of flat $SL(2, \mathbb{C})$ connections in the Hitchin moduli space in Exercise 1e).
5. Combine the results of Exercises 2, 3, and 4 to find the mirror branes $\tilde{\mathcal{B}}$ in each case.

6. In the context of Exercise 1a), find the mirror of the (B, A, A) brane \mathcal{B} supported on the base (resp. the fiber) of the Hitchin fibration. In each case, verify that the resulting brane is indeed of type (B, B, B) , that is hyper-holomorphic.
7. In each case (b) – (e) of Exercise 1, describe (B, A, A) brane supported on a “real slice” \mathcal{B} , that is supported on $\mathcal{M}_{\text{flat}}(G_{\mathbb{R}}, \Sigma)$ where $G_{\mathbb{R}}$ is either $SU(2)$ or $SL(2, \mathbb{R})$.
8. Combine the results of Exercises 2 and 7 to find the mirror (B, B, B) branes $\tilde{\mathcal{B}}$ in each case.

2 Bibliography

The lectures will be based on:

1. S. Gukov, Takagi Lectures: *Quantization via Mirror Symmetry*, Japan. J. Math. **6** (2011) 65119, arXiv:1011.2218.