



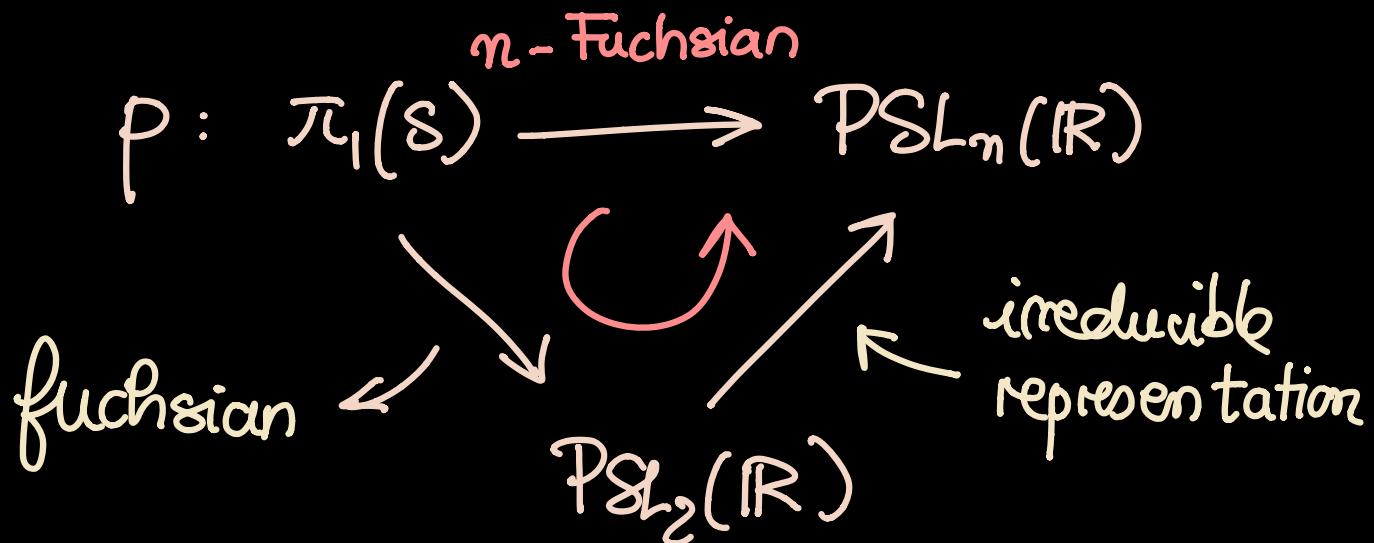
François LABOURIE

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Université Côte d'Azur

Hitchin representation

$S =$ closed connect oriented surface genus $g \geq 2$.



A Hitchin representation is
a representation that can be
deformed in an n -Fuchsian

Hitchin component

$\mathcal{H}(n) = \{ \text{Hitchin representation} \}$

connected component of $\text{Rep}(\pi_1(S), \text{PSL}_n(\mathbb{R}))$

$\mathcal{T}(S)$ = Teichmüller space

Hitchin representations

Nice dynamical properties

- ↪ associated to a “geodesic flow”
- ↪ length spectrum, entropy etc...
- ↪ Mapping class group acts properly on $\partial(\mathbb{H}^n)$
- ↪ $\forall \gamma \in \pi_1(S) \setminus \{\text{id}\}$, $\rho(\gamma)$ is \mathbb{R} -split

Hitchin component

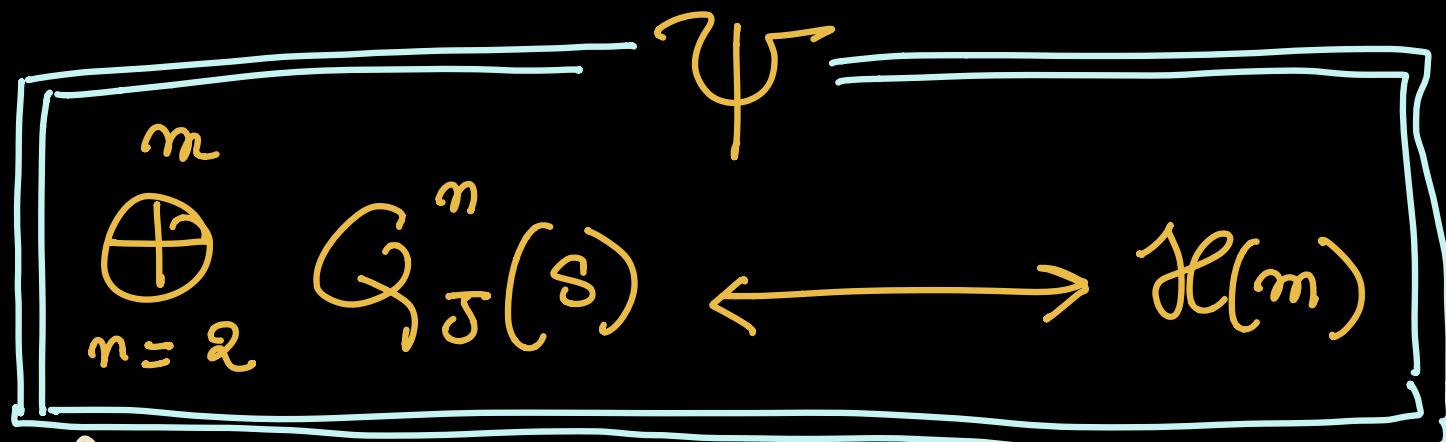
behaves a lot like Teichmüller component $\mathfrak{sl}(2)$ from the dynamical point of view.

What about uniformisation ?

Hitchin parametrisation

$X = (S, \mathcal{J})$ be a Riemann surface

$\mathcal{G}_{\mathcal{J}}^n(S) := \{ n - \text{holomorphic diff on } X \}$



break the Mapping class group symmetry

$\det \mathcal{E} =$ bundle of Teichmüller
 $\mathcal{E}_J = \bigoplus_{n=3}^m Q_J^n(s)$

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We have a MCG equivariant

$\Phi : \mathcal{E} \rightarrow \mathfrak{sl}(n)$

$\nwarrow \nearrow$

same dimension

- $\mathcal{E} = \{\text{space of equivariant minimal surface in } M_m \text{ under Hitchin representations}\}$
- \mathcal{E} has a Kähler structure
[I. Kim - G. Zhang, L]
- Φ is surjective

Conjecture

| is Φ injective ? |

is there a complex interpretation of $\mathcal{H}(n)$?

is $\mathcal{H}(n)/M$ a vector bundle over Riemann moduli space?

(J.W with Samborino)

Hitchin representation

↪ geodesic current = a locally finite
 $\pi_1(S)$ -invariant measure on $\partial_\infty \pi_1(S)^2$
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If X = Riemann surface

$$\langle v, X \rangle_m(z) := \int_{(\partial_\infty \mathbb{H})^2} \left[\frac{1}{(z-x)} - \frac{1}{(z-y)} \right] dv(x,y) dz^m$$

Given γ there is a unique β so
that $\langle \gamma, \beta \rangle_2(z) = 0$

$\mathcal{H}(n)$ \longrightarrow Teich

Is this a fibration? are the
fibers contractible? what are
the meanings of $\langle \gamma | \beta \rangle_m$?

Then :

is there a complex interpretation of $\mathcal{H}(n)$?

—
is $\mathcal{H}(n)/_{MCG}$ a vector bundle over Riemann moduli space?

Fock-Thomas higher complex structures.

A question (not unrelated)

Let M be a manifold, i a function on $M \times M$ so that

(x, x) is a minimum of $y \rightarrow i(x, y)$
assume $i(x, y) = i(y, x)$

Say i is W if it enjoys the following property :

the pairing $T_x M \times T_y M \rightarrow \mathbb{R}$

$(u, v) \mapsto D_y[(\partial_x i(x, y)) u] v$ is non

degenerate. To be more precise. Let :

let $f^y: x \mapsto i(x, y)$

let $g^u: y \mapsto D_x f^y(u)$

And $\langle u | v \rangle = D_y g^u(v)$. $\rightarrow i = \text{intersection}$

The question is does i has the W property?

that would imply a new parametrisation of Σ by quadratic diff.

